Damping of Very Soft Moving Quarks in High-Temperature QCD

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(February 1, 2008)

Abstract

We determine the analytic expression of the damping rates for very soft moving quarks in an expansion to second order in powers of their momentum in the context of QCD at high temperature. The calculation is performed using the hard-thermal-loop-summed perturbation scheme. We describe the range of validity of the expansion and make a comparison with other calculations, particularly those using a magnetic mass as a shield from infrared sensitivity. We discuss the possible occurrence of infrared divergences in our results and argue that they are due to magnetic sensitivity.

pacs: 11.10.Wx 12.38.-t 12.38.Bx 12.38.Mh

keywords: hard thermal loops. soft quark damping. infrared sensitivity.

ENSK–TP–18

I. INTRODUCTION

One runs into difficulties when one applies standard loop expansion to gauge theories at high temperature T: physical quantities like the dispersion laws become gauge dependent. Early work on the QED plasma using a hydrodynamic approach is [1], followed by [2], taking account of one-loop quantum effects. The work [3] discusses to one-loop order the QCD polarization tensor at high temperature and quark density and determines the gluon

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dispersion laws to lowest order in the coupling g. It shows that these dispersion laws are gauge invariant but the one-loop-order gluon damping rates in the long-wavelength limit are not. It also shows that while chromoelectric Debye screening does occur to lowest order in the one-loop calculation, chromomagnetic screening does not, a gauge invariant statement. The non-screening of chromomagnetic fields at lowest order is also discussed in [4–6]. The massless-quark spectrum to lowest order in the coupling is described in [7] and the full quasiparticle spectra to lowest order at high T for the whole momentum range are given in [8]. The quasiparticle spectra are also described in [9] for gluons and [10] for quarks.

The problem of gauge dependence of the damping rates has been emphasized in works in which the gluon damping rates, particularly at zero momentum, have been calculated to one-loop order in various gauges and schemes and different results have been obtained [11]. It was realized that the problem is related to the fact that at high temperature, higher-loop diagrams can contribute to lower orders in powers of the coupling [12]. In other words, the standard loop expansion is no more an expansion in powers of g^2. In a series of papers, Braaten and Pisarski developed a systematic method for an effective perturbative expansion that sums the so-called hard thermal loops (HTL) into effective propagators and, equally important for gauge theories, effective vertices [13–15]. Using this method, the transverse-gluon damping rate $\gamma_t(0)$ at zero momentum was shown to be finite, positive and independent of the gauge [16]. Later, a generating-functional formalism for high-T QCD in the HTL approximation was developed [17] and a relation to the eikonal of a Chern-Simons gauge theory was found [18]. From there, a hydrodynamic approach showed that the HTL approximation is essentially 'classical' [19].

Once developed, the important question to answer is whether the HTL-summed perturbation is reliable for calculations in QCD at high temperature. If so, it would constitute an adequate framework for describing the properties of the quark-gluon plasma. Of particular interest is the question of infrared sensitivity in massless gauge theories, worsened at finite temperature by the presence of the Bose-Einstein distribution which behaves like $1/k$ for very small gluon energies k: quantities tend to diverge like powers of the infrared cutoff rather than logarithmically as is the case at zero temperature [20]. This infrared problem is prior to the advent of the HTL scheme. It is for example shown in [21] that infrared (and mass) singularities do occur but cancel out in first-order radiative corrections to the production of lepton pairs in thermal (massless) QCD. It is therefore most interesting to see if the HTL scheme is of any help in this regard. More precisely, does the HTL-summed perturbation constitute a workable framework in which infrared divergences are cured consistently order by order in the coupling?

It turns out that the HTL summation dresses the massless quarks and gluons allowing them to acquire thermal masses of order gT, m_f and m_g respectively [3,7–10]. This means that to this lowest order gT in effective perturbation, the infrared region is 'safe'. But as

recalled, static chromomagnetic fields do not screen at this lowest order, they are believed to do so at the next-to-leading order g^2T, the so-called magnetic scale [4–6]. Therefore, the test for the infrared safeness of the HTL-summed expansion starts really in next-to-leading order calculations.

Much work has been carried regarding the use of the HTL summation up to next-to-leading order. For example, it is shown in [22] that the HTL summation cures the infrared logarithmic divergence in the production rate of hard thermal photons in high-temperature QCD with massless quarks. Also, the damping of fast moving fermions has been thoroughly investigated in [12,23,20,24,25] for hot QCD and [26,27] for hot QED. In [26] for instance, it is first performed a bare (i.e., not HTL-summed) two-loop calculation and by taking the fermion slightly off-shell, there occurs to this order cancellation of infrared, both electric and magnetic, singularities. It is then shown that the dominant on-shell graphs are those dictated by the HTL summation. Also, it is shown in [20] that it is in fact possible to find the leading contribution $g^2T\ln 1/g$ to the damping rates for energetic (hard) quarks and gluons in high-temperature QCD without having recourse to the full machinery of the HTL-summation program, a leading contribution already obtained for quarks in [12], see also [23].

It seems therefore that the infrared problem can somewhat be brought under control when it comes to describing fast-moving (hard) quasiparticles. But how about slow-moving (soft) quarks and gluons, particularly those on shell? A first indication of the sensitivity of the HTL perturbation to slow moving particles can be found in [28] where the production of non-thermalized soft real photons in HTL-summed perturbation in high-T QCD is discussed. It is argued that this scheme fails to screen mass singularities in that it is not able to yield a finite contribution to leading order to the production rate, a physical quantity. However, the divergences involved in [28] are collinear in nature and come from dressed vertices. In this regard, an improved action has been proposed in [29] which incorporates an asymptotic mass m_∞ that removes singularities coming from light-like external momenta.

As to the importance of the magnetic sector and the infrared sensitivity of the HTL scheme at next-to-leading order calculations, this is well demonstrated in the works [30–32]. Indeed, [30] calculates in HTL-summed perturbation the non-abelian Debye mass at next-to-leading order from the static limit of the polarization tensor. [31] determines the same physical quantity at the same order in the same scheme, but from the correlator of two Polyakov loops, a gauge invariant quantity. Paper [32] discusses the more general problem of next-to-leading order non-abelian Debye screening in one-loop HTL-summed perturbation. It argues that since the magnetic sector is nonperturbative in essence, the perturbative next-to-leading-order results may not be reliable. This is explicitly shown in, for example, the strong dependence of the analytic structure of the inverse of the static longitudinal propagator on the infrared behavior of the transverse gluons where the results differ significantly

depending on whether we regularize the infrared sector by introducing a magnetic mass[1] or not. One other interesting point discussed is the important role of the magnetic mass in cancelling the gauge dependent terms when obtaining Debye screening from the Polyakov-loop correlator. Finally, comparison with lattice simulations indicates that the magnetic-mass enhanced results are more compatible with the lattice ones, hence the importance of the magnetic sector.

However, Debye screening is static in nature. It is therefore interesting to examine dynamic on-shell quantities in order to understand better the infrared behavior of hot gauge theories in HTL-summed perturbation for soft moving quasiparticles. It turns out that the damping rates at lowest order for such (very) soft moving quasiparticles are quite suitable. Indeed, the HTL quasiparticle self-energies are real and so no damping is manifest at order gT; it starts at precisely the magnetic scale g^2T. Thus, to exhibit damping, one needs to add to the inverse propagators the next-to-leading order contributions to the self-energies which are, *as dictated* by the HTL-summed expansion, one-loop corrections with soft loop momenta, hence all propagators and vertices have to be HTL dressed.

The first such calculation is paper [16] which determined the damping rate $\gamma_t(0)$ for transverse on-shell gluons with zero momentum and found:

$$\gamma_t(0) = 0.088 N_c\, g^2 T, \tag{1}$$

where N_c is the number of colors. The analytic calculation of $\gamma_t(p)$ to order $(p/m_g)^2$ where p is the momentum of the very soft gluon was carried in [33]. The zeroth order (1) was recovered and it was indicated that the coefficient of the order $(p/m_g)^2$ may carry infrared divergences. The infrared sensitivity of the on-shell gluon damping rates has been emphasized in [34,35] where the damping rate $\gamma_l(0)$ for longitudinal gluons with zero momentum was determined to lowest order g^2T and found to be different from $\gamma_t(0)$ and infrared divergent. This is to be contrasted with the fact that at zero momentum, there must be no difference between longitudinal and transverse gluons [16]. This statement is emphasized in [36] where a Slavnov-Taylor identity for the gluon polarization tensor in Coulomb gauge is derived and when applied to the next-to-leading order gluon self-energy, the equality $\gamma_l(0) = \gamma_t(0)$ is obtained[2].

The next step must be the discussion of the damping rates for very soft moving quarks at lowest order g^2T. This is because quarks are also important in the structure of hot QCD.

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[1]Introduced as an infrared regulator, a point we come to later in section three.

[2]It is assumed in [36] that the spatial next-to-leading order HTL-summed gluon self-energy is isotropic at zero momentum. Our explicit and direct calculations do not recover this isotropy. This issue will be addressed in detail in [37].

Since they too acquire a thermal mass m_f at order gT and their damping rates start at the magnetic scale g^2T, it is all but legitimate to inquire about their infrared sensitivity. There are already the two works [38] and [39] which determined independently the damping rates $\gamma_\pm(0)$ for quarks with zero-momentum and found:

$$\gamma_\pm(0) = a_0 C_f \, g^2 T, \tag{2}$$

where $C_f = (N_c^2 - 1)/2N_c$ and a_0 is a finite constant depending on N_c and N_f, the number of flavors. For example, for $N_c = 3$ and $N_f = 2$, we have $a_0 = 0.111\ldots$, [38,39]. Result (2) for quarks resembles result (1) for gluons. It is therefore interesting to determine the damping rates for *moving* quarks, but with *very* soft momenta. In this article, we attempt to obtain to lowest order g^2T an analytic expression for the damping rates $\gamma_\pm(p)$, where $\mathbf{p} = p\widehat{\mathbf{p}}$ is the momentum of the quark, using *only* as ingredients what the HTL-summed expansion dictates.

Obtaining a compact expression for $\gamma_\pm(p)$ would be ideal but hardly feasible technically. Rather, we attempt to obtain an expression for $\gamma_\pm(p)$ in powers of p/m_f up to second order. This expansion is carried early on in the calculation. We will specify in section three its range of validity and argue that in order to get an explicit expression for the damping rates, manipulating otherwise is practically intractable. In this work, we describe in detail how we obtain the analytic expressions for the first three coefficients a_0, a_1 and a_2 involved in the expansion, see (42), and we defer the numerical evaluation of these to future work [40]. This is because it necessitates the extraction of the potentially infrared-divergent pieces from the finite contributions, something somewhat complicated. An additional complication comes from the presence of two soft masses, m_g and m_f, and the discussion necessitates working each case apart. The numerical evaluation also necessitates the handling of potential divergences coming from soft light-like loop momenta. Experience with the transverse gluon damping rate $\gamma_t(p)$ [37] indicates that these latter divergences may ultimately be brought under control, but the infrared ones would most likely persist. One interesting aspect to mention is that the order p/m_f in $\gamma_\pm(p)$ does not vanish contrary to the gluonic case. Preliminary results [40] tend to indicate that there are no infrared divergences in the coefficient[3] a_1 but they tend to appear in a_2. On the one hand, this puts 'some water' into our arguments regarding the acceptability of our early expansion in powers of the very soft external momentum. On the other, recalling that the infrared divergences we find in the gluonic sector start also at order $(p/m_g)^2$ [34,35,37], it would be interesting to try to understand why this is so.

Finally, it is useful to recall that the intensive use of the damping rates as a mean to

[3]We already know from [38,39] that the first coefficient a_0 is safe.

probe the properties of finite-T gauge theories is also due to the fact that in general, calculations in the full HTL-summed perturbation are quite difficult beyond lowest order. The damping rates are the simplest such non-trivial quantities to handle. Indeed, though they come from one-loop graphs with dressed propagators and vertices, they are defined through the imaginary part of the effective self-energies, something that simplifies significantly the calculation. The only attempt to correct in HTL-summed perturbation the quasiparticle spectra to order g^2T we are aware of is that of [41].

This paper is organized as follows. In the next section, we recall the definition of the quark damping rates and write them in the context of the HTL-summed perturbation. Their determination amounts to that of the imaginary part of the next-to-leading order quark self-energy which we carry in detail in section four. Section three is devoted to discussing the expansion of the effective self-energy in powers of the external momentum and to the manner with which we regularize the infrared region. A comparison with other computational and regularization schemes is also carried, most particularly those shifting the pole of the effective gluon propagators with a magnetic mass. The final results are presented and discussed in the last section.

II. QUARK DAMPING RATES IN HTL-SUMMED PERTURBATION

We use the imaginary-time formalism in which the euclidean momentum of the quark is $P^\mu = (p_0, \mathbf{p})$ such that $P^2 = p_0^2 + p^2$ with the fermionic Matsubara frequency $p_0 = (2n+1)\pi T$, n an integer. Real-time amplitudes are obtained via the analytic continuation $p_0 = -i\omega + 0^+$ where ω is the energy of the quark. A momentum is said to be soft if both ω and p are of order gT; it is said to be hard if one is or both are of order T. The *three*-momentum $\mathbf{p}$ of the on-shell quark is said to be *very* soft if p is much smaller than gT, say of the order g^2T and smaller. We follow closely the notation of [13] and the HTL results we quote in this section can all be found there, see also [14,15].

The effective propagator for the quark can be written as:

$$^*\Delta_F(P) = -\left[\gamma_{+p}\Delta_+(P) + \gamma_{-p}\Delta_-(P)\right], \tag{3}$$

where γ^μ are the euclidean Dirac matrices, $\gamma_{\pm p} = (\gamma^0 \pm i\gamma.\hat{\mathbf{p}})/2$ and $\Delta_\pm = (D_0 \mp D_s)^{-1}$ with:

$$D_0 = ip_0 - \frac{m_f^2}{p} Q_0\left(\frac{ip_0}{p}\right);$$
$$D_s = p + \frac{m_f^2}{p}\left[1 - \frac{ip_0}{p} Q_0\left(\frac{ip_0}{p}\right)\right], \tag{4}$$

where the quark thermal mass is $m_f = \sqrt{C_f/8}\, gT$ and $Q_0(x) = \frac{1}{2}\ln\frac{x+1}{x-1}$. The poles of $\Delta_\pm(-i\omega, \mathbf{p})$ determine the dispersion laws[4] $\omega_\pm(p)$ to lowest order in g. For soft quarks, one has:

$$\omega_\pm(p) = m_f\left[1 \pm \frac{p}{3m_f} + \frac{1}{3}\left(\frac{p}{m_f}\right)^2 \mp \frac{16}{135}\left(\frac{p}{m_f}\right)^3 + \frac{1}{54}\left(\frac{p}{m_f}\right)^4 \right.$$
$$\left. \pm\frac{32}{2835}\left(\frac{p}{m_f}\right)^5 - \frac{139}{12150}\left(\frac{p}{m_f}\right)^6 \pm \ldots\right]. \tag{5}$$

At this lowest order gT, $\omega_\pm(p)$ are real and the quarks are not damped. To get the damping rates to their lowest order, one has to include in the dispersion relations the contribution from the effective quark self-energy ${}^*\Sigma(P)$. Therefore, the inverse of the quark propagator becomes:

$$\Delta_F^{-1}(P) = {}^*\Delta_F^{-1}(P) - {}^*\Sigma(P)\,. \tag{6}$$

The effective quark self-energy has also the decomposition ${}^*\Sigma = \gamma^0\, {}^*D_0 + i\gamma.\hat{\mathbf{p}}\, {}^*D_s$ where ${}^*D_0$ and ${}^*D_s$ are the two functions to be determined in HTL-summed perturbation. The inverse of the quark propagator is then:

$$\Delta_F^{-1}(P) = -\left[\gamma^0\left(D_0 + {}^*D_0\right) + i\gamma.\hat{\mathbf{p}}\left(D_s + {}^*D_s\right)\right]. \tag{7}$$

The damping rates for quarks are $\gamma_\pm(p) \equiv -\mathrm{Im}\Omega_\pm(p)$ where $\Omega_\pm$ are the poles of $\Delta_F(-i\Omega, \mathbf{p})$. Since the self-energy ${}^*\Sigma$ is g-times smaller than ${}^*\Delta_F^{-1}$, we have to lowest order:

$$\gamma_\pm(p) = \left.\frac{\mathrm{Im}\, {}^*f_\pm(-i\omega, p)}{\partial_\omega f_\pm(-i\omega, p)}\right|_{\omega=\omega_\pm(p)+i0^+}, \tag{8}$$

where $f_\pm = D_0 \mp D_s$, ${}^*f_\pm = {}^*D_0 \mp {}^*D_s$ and ∂_ω stands for $\partial/\partial\omega$. Using the expressions in (4), it is easy to expand the denominator in the above relation in powers of p/m_f. One then obtains:

$$\gamma_\pm(p) = \frac{1}{2}\left[1 \pm \frac{2}{3}\frac{p}{m_f} - \frac{2}{9}\left(\frac{p}{m_f}\right)^2 + \ldots\right] \left.\mathrm{Im}\, {}^*f_\pm(-i\omega, p)\right|_{\omega=\omega_\pm+i0^+}. \tag{9}$$

We see that determining $\gamma_\pm(p)$ to lowest order in g amounts to calculating the imaginary part of the next-to-leading order quark self-energy.

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[4] (+) for real quarks and (−) for 'plasminos' [42], only thermally excited quasiparticles.

The HTL-summed perturbation [13–15] dictates that the next-to-leading order quark self-energy is given in imaginary-time formalism by:

$$ {}^*\Sigma\,(P) = {}^*\Sigma_1\,(P) + {}^*\Sigma_2\,(P)\,, \tag{10} $$

where we have:

$$ {}^*\Sigma_1\,(P) = -g^2 C_f \mathrm{Tr}_{\mathrm{soft}} \left[{}^*\Gamma^\mu\,(P,-Q;-K)\ {}^*\Delta_F\,(Q)\ {}^*\Gamma^\nu\,(-P,Q;K)\ {}^*\Delta_{\mu\nu}\,(K)\right], \tag{11} $$

and:

$$ {}^*\Sigma_2\,(P) = -\frac{i}{2} g^2 C_f \mathrm{Tr}_{\mathrm{soft}} \left[{}^*\widetilde{\Gamma}^{\mu\nu}\,(P,-P;K,-K)\ {}^*\Delta_{\mu\nu}\,(K)\right]. \tag{12} $$

K is the soft gluon loop momentum, $Q = P - K$ and $\mathrm{Tr} \equiv T \sum\limits_{k_0} \int \frac{d^3k}{(2\pi)^3}$ with $k_0 = 2n\pi T$, a bosonic Matsubara frequency. The subscript "soft" means that only soft values of K are allowed in the integrals; hard values have dressed the propagators and vertices. Note that since the loop momentum K is soft, both propagators and vertices involved in (10) must be dressed.

The effective gluonic propagator ${}^*\Delta_{\mu\nu}\,(K)$ is taken in the strict Coulomb gauge where it has a simplified structure. It is given by ${}^*\Delta_{00}\,(K) = {}^*\Delta_l\,(K)$, ${}^*\Delta_{0i}\,(K) = 0$ and ${}^*\Delta_{ij}\,(K) = \left(\delta_{ij} - \widehat{k}_i\widehat{k}_j\right)\ {}^*\Delta_t\,(K)$ with ${}^*\Delta_l$ and ${}^*\Delta_t$ having the following expressions:

$$ {}^*\Delta_l\,(K) = \frac{1}{k^2 - \delta\Pi_l\,(K)}\,; \qquad\qquad {}^*\Delta_t\,(K) = \frac{1}{K^2 - \delta\Pi_t\,(K)}\,, \tag{13} $$

where $\delta\Pi_l(K) = 3m_g^2 Q_1(\frac{ik_0}{k})$ and $\delta\Pi_t(K) = \frac{3}{5}m_g^2\left[Q_3(\frac{ik_0}{k}) - Q_1(\frac{ik_0}{k}) - \frac{5}{3}\right]$. $Q_i(\frac{ik_0}{k})$ is a Legendre function of the second kind and the gluon thermal mass $m_g{=}\sqrt{N_c + N_f/2}\,gT/3$. The effective (dressed) vertices ${}^*\Gamma$ intervening in (10) are of the form:

$$ {}^*\Gamma = \Gamma + \delta\Gamma\,, \tag{14} $$

where Γ is the bare (tree) vertex and $\delta\Gamma$ is the corresponding hard thermal loop. The two effective vertices that enter the calculation of the effective self-energy (10) are the effective quark-gluon vertex:

$$ {}^*\Gamma^\mu(P,Q;R) = \gamma^\mu + m_f^2 \int \frac{d\Omega_s}{4\pi} \frac{S^\mu \not{S}}{PS\,QS}\,, \tag{15} $$

where the second term is the hard thermal loop, and the effective two-gluons-quark-antiquark vertex:

$$ {}^*\widetilde{\Gamma}^{\mu\nu}(P,-P;K,-K) = -2m_f^2 \int \frac{d\Omega_s}{4\pi} \frac{S^\mu S^\nu \not{S}}{PS\,(P+K)S\,(P-K)S}\,. \tag{16} $$

Note that the bare two-gluons-quark-antiquark vertex is zero so that the corresponding effective vertex is just the hard thermal loop. In both (15) and (16), $S \equiv (i, \hat{\mathbf{s}})$ and Ω_s is the solid angle of $\hat{\mathbf{s}}$.

The task is to attempt to get an expression for the imaginary part of the effective quark self-energy ${}^*\Sigma\,(P)$. The 'natural' sequence of steps to follow is first to perform the angular integrations in the dressed vertices (15) and (16). Next is to do the Matsubara sum in (10). Only then the continuation to real quark energies $p_0 = -i\omega + 0^+$ can be taken and the on-shell condition enforced. Last is to find a way to perform the integration over the gluon loop three-momentum $\mathbf{k}$. However, given the complicated expressions we are faced with, it is practically very difficult to follow this sequence of operations. What we do in this work is first expand the effective self-energy in powers of the quark momentum p/m_f. This allows for an easy angular integration over Ω_s. Only then do we perform the Matsubara sum, this by using the spectral representation of the different quantities involved. The angular integration over Ω_k is subsequently done and the remaining integrals can be calculated numerically. We discuss this procedure in more detail in the next section.

III. REGULARIZATION AND EXPANSION IN EXTERNAL MOMENTUM

The expansion in powers of the external momentum of the HTL-summed next-to-leading order self-energies can be questioned from the outset in view of the fact that infrared divergences do appear in next-to-leading order physical quantities like the damping rates [35]. Are these divergences genuine or merely artifacts due to the method used? First, recall that we are considering only very soft external momenta. Note also that the HTL framework itself allows for an expansion of quantities in powers of the soft external momenta. For example, the gluon and quark on-shell energies $\omega(p)$ are obtained in the literature in the form of a series in powers of soft p [43,15]. The same is true for the residue and cut functions intervening in the spectral decomposition of the effective propagators [35]. It is therefore legitimate to expect the perturbation *built on* hard thermal loops (these being considered as a zeroth order approximation) to be analytic in very soft p, and hence admit an expansion in powers of such momenta. Also, such an expansion is not proper to this work, it has been previously used in the literature, for example in [44].

There is of course a distinction between the analyticity in p and that in g. For example, the standard loop expansion of QCD is in powers of g^2 whereas in HTL-summed perturbation, the expansion is in powers of $\sqrt{g^2}$. The same is true for other theories like the prototype $\lambda\phi^4$ theory [46] and QED [47]. This may introduce a non-analyticity with respect to g in some quantities, but does not necessarily change drastically the analytic behavior of these quantities with respect to very soft p. Take for example the estimation of the soft gluon damping rates made in [45]:

$$\gamma_{t,l}(p) \sim -\frac{g^2 N_c T}{4\pi} \ln g\, v_{t,l}(p), \tag{17}$$

where $v_{t,l}(p)$ are the corresponding group velocities. These rates are clearly non-analytic in small g, but perfectly analytic in p: they even tend to zero[5] as $p \to 0$.

Regarding the damping rates, our starting position is that the quark-gluon plasma is to be a stable phase of hadronic matter, at least for very soft excitations [48]. QCD at high temperature in the (lowest-order) HTL approximation is 'finite'. At next-to-leading order, HTL-summed perturbation yields finite and positive damping rates for zero-momentum on-shell quarks and transverse gluons. The stability criterion ensures that we must expect the damping rates to remain finite and positive for non-zero very soft momenta. This translates into expecting the damping rates to admit a series expansion in powers of these very soft external momenta. This of course does not rule out a possible loss of analyticity for larger values of p, even just soft values. That would simply indicate new physics to explore. But because of the stability criterion, the analyticity must be preserved for very soft momenta. This is one important check to use in order to discuss the consistency and completeness of a given calculational scheme like the HTL-summed perturbation.

Expecting an infrared problem, we introduce an infrared cut-off $\eta > 0$ such that $\int_0^{+\infty} dk$ in (11) and (12) is replaced by $\int_\eta^{+\infty} dk$. The cut-off η is fixed for the rest of the calculation. It is physically useful to see it as representing the magnetic scale g^2T. This means that k is never smaller than η. In other words, we are summing contributions from all soft momenta[6] k but not the very soft ones, i.e., those smaller than η. We *always* regard the external momentum p as smaller than η. We are therefore always working in the kinematic region $0 \leq p < \eta \leq k$. This allows for the expansion in powers of p of all quantities that are functions of $q = |\mathbf{p} - \mathbf{k}|$. This is true in particular for $1/QS$ and the effective propagators ${}^*\Delta\,(Q)$. The expansion of $1/PS$ does not pose a problem in itself.

It is useful to emphasize once more that our calculation sums the contributions from *only* the soft integration momenta $\eta \leq k$: the very soft momenta $0 \leq k < \eta$ are systematically excluded and the hard region is cut by the spectral densities [35]. If the integration is not sensitive to the very soft region (the magnetic sector), then the subsequent limit $\eta \to 0$ in the final result should be smooth. If on the contrary there is sensitivity, it would mean that important contributions from this sector may be 'missed' by the HTL-summed perturbation. This is the essence of our point. Indeed, recall that regarding the self-energies, the HTL scheme discusses the two scales T (hard) and gT (soft), whereas with g and T, one has a hierarchy of scales g^nT with n a nonnegative integer. As a matter of fact, [49] argues that

[5] A non-acceptable limit as we will discuss shortly.

[6] Hard momenta are already summed in the hard thermal loops.

there may be scales between T and gT that play a significant role, and so n may not even be an integer. Since the HTL scheme, when built, does not consider effects like magnetic screening which (are believed to) arise nonperturbatively at order g^2T and are not present at the HTL level, a perturbation built on the HTL summation may not be able to reproduce them. What it can do is to bring what contributes from the soft region to the very soft one. Therefore, excluding the very soft region from the k-integration as we do may not be all unreasonable a thing to do. We think that the presence of magnetic sensitivity which manifests itself in infrared divergences indicates the very presence of these magnetic effects that the HTL scheme seems to be not able to accommodate.

Let us compare our calculation of the damping rates with the estimation (17) mentioned above. This latter is different in many respects from the one we carry. It is obtained in the kinematic region where the loop momentum k is restricted to the very soft region ($0 \leq k < \eta$ in our notation) whereas p is just soft, of order gT [45]. In some sense, there, it is the very soft momenta that are integrated out; the soft ones are disregarded, something opposite to what we do. Result (17) cannot be carried to the very soft region $p < \eta$, in particular to the point $p \to 0$ for it will give zero (using the expressions of the group velocities at very soft momenta) whereas the damping rates there are finite. At the same time, our results can never be carried to the region $p > \eta$. Clearly then, it should not be problematic if different analytic results are obtained. In fact, if really different results are obtained, which is the case, it only constitutes a further indication of the sensitivity of the HTL scheme to the magnetic sector. Equally interesting to note in the estimation (17) is that, in order to screen the divergent behavior at very soft momenta k, a regularization is used, which amounts to introducing a magnetic mass $m_{\rm mag}$ in the otherwise divergent propagators. It is clear that screening of chromomagnetic fields, if it occurs, is not necessarily going to manifest itself by a simple shift of the pole in the corresponding propagator by a momentum independent magnetic mass [32].

The presence of magnetic effects not handled by the HTL scheme is discussed in [44]. It is argued there that for distances to order $1/T$, ordinary perturbation (the standard loop expansion) is reliable. For distances to order $1/gT$, the effective theory that screens static chromoelectric fields (the Braaten-Pisarski scheme) is reliable and it can be treated in perturbation. However, for distances to order $1/g^2T$, one needs another effective theory which cannot be treated by perturbation (treated by lattice simulations for example). This last statement is emphasized by comparing the asymptotic behavior of the Polyakov-loop correlator determined from a magnetic lagrangian with that determined from an electric lagrangian with a put-by-hand magnetic mass $m_{\rm mag}$. With the magnetic lagrangian, an exponential decay governed by the lowest glueball state is obtained, an asymptotic behavior different from the one obtained from the magnetic-mass enhanced electric lagrangian. A comparison of these results with lattice simulations indicates that the glueball-state result

is more compatible with the lattice ones. One interesting inference one can draw from the above comments is that regularizing the infrared sector in HTL perturbation with a simple magnetic mass may not be the best description of magnetic effects, in particular if those are not incorporated in the scheme itself.

It is important to stress that from a pure computational standpoint, matters are not straightforward if we defer in the self-energies the expansion in p after the angular integrals. Indeed, one has to deal with expressions quite complicated and involved, something of the sort $T\sum_{k_0}\int d^3k\int d\Omega_{s_1}\int d\Omega_{s_2}\frac{f(\widehat{\mathbf{s}}_1,\widehat{\mathbf{s}}_2)}{PS_1\,KS_1\,PS_2\,QS_2}\,{}^*\Delta\left(K\right)\,{}^*\Delta\left(Q\right)$. The Matsubara sum can be performed if the effective propagators, $\frac{1}{KS_1}$ and $\frac{1}{QS_2}$ are replaced by their respective spectral decomposition, see below. But this will bring in more than one energy denominator, which would compromise the straightforward extraction of the imaginary part of the effective self-energy. More serious a problem is the subsequent angular integration which will be very difficult, it not impossible, to perform [14].

Finally, it turns out that for quarks, the expansion is in powers of p/m_f and not in $(p/m_f)^2$, as is the case for gluons (where m_f is replaced by m_g), [33–35]. Preliminary results [40] tend to indicate that the second coefficient (that of p/m_f) is infrared safe together with the first one. This may suggest then that the expansion in powers of p is not sole to 'blame' for obtaining infrared divergent damping rates; other effects may be in play.

IV. IMAGINARY PART OF ONE-LOOP HTL-SUMMED QUARK SELF-ENERGY

Now we present the calculation of the HTL-summed next-to-leading order quark self-energy from which we extract the imaginary part. We first describe how we get an expression for $\mathrm{Im}\,{}^*\Sigma_1\left(P\right)$ defined in (11) and then for $\mathrm{Im}\,{}^*\Sigma_2\left(P\right)$ defined in (12). From now on, we take $m_f=1$. This will simplify the final expressions we obtain. There remains another soft mass in the problem, m_g, and so we define $m=m_g/m_f=\frac{4}{3}\sqrt{\frac{N_c(N_c+N_f/2)}{N_c^2-1}}$. It is easy to see that we always have $m>1$.

A. Calculation of $\mathrm{Im}\,{}^*\Sigma_1\left(P\right)$

Using the structure of the fermion propagator (3) and that of the gluon propagator in the strict Coulomb gauge given just before (13), we see that ${}^*\Sigma_1\left(P\right)$ is composed of four terms:

$$\begin{aligned}{}^*\Sigma_1(P)=\frac{8}{T^2}\sum_{\varepsilon=\pm}T\sum_{k_0}\int\frac{d^3k}{(2\pi)^3}\Big[&{}^*\Gamma^0\left(P,-Q;-K\right)\Delta_\varepsilon\left(Q\right)\gamma_{\varepsilon q}\,{}^*\Gamma^0\left(-P,Q;K\right)\,{}^*\Delta_l\left(K\right)\\&+{}^*\Gamma^i\left(P,-Q;-K\right)\Delta_\varepsilon\left(Q\right)\gamma_{\varepsilon q}\,{}^*\Gamma^j\left(-P,Q;K\right)\,(\delta_{ij}-\widehat{k}_i\widehat{k}_j)\,{}^*\Delta_t\left(K\right)\,.\end{aligned}\tag{18}$$

The first two terms denoted ${}^*\Sigma_{\varepsilon l}(P)$, those with the longitudinal gluon propagator, are calculated separately from the two others denoted ${}^*\Sigma_{\varepsilon t}(P)$. We will illustrate the different steps of the calculation for ${}^*\Sigma_{-l}(P)$. Using the definition of the effective vertex (15) and making the change of integration variable $K \to P - K$, we have:

$$
\begin{aligned}
{}^*\Sigma_{-l}(P) = \frac{8}{T^2}\, T\widetilde{\sum_{k_0}} \int \frac{d^3k}{(2\pi)^3} \Bigg[\gamma_{+k} - \int \frac{d\Omega_s}{4\pi} \, \frac{2i\not{S} + \gamma^0 \gamma.\widehat{\mathbf{k}}\,\not{S} + \not{S}\gamma.\widehat{\mathbf{k}}\gamma^0}{2PS\,KS} \\
- \int \frac{d\Omega_{s_1}}{4\pi} \int \frac{d\Omega_{s_2}}{4\pi} \frac{\not{S}_1\, \gamma_{-k}\, \not{S}_2}{PS_1\, KS_1\, PS_2\, KS_2} \Bigg] \, \Delta_-\left(K\right)\, {}^*\Delta_l\left(Q\right) . \qquad (19)
\end{aligned}
$$

The tilde over the sum sign indicates that k_0 is fermionic.

Let us start with I_1, the term in (19) where there is one solid-angle integral over Ω_s. This latter is carried in a reference frame where $\widehat{\mathbf{k}}$ is the principle axis (i.e., the 'z-axis'). The solid angle is then $\Omega_s = (\theta, \varphi)$ such that $\widehat{\mathbf{k}}.\widehat{\mathbf{s}} = \cos\theta$ and $\widehat{\mathbf{p}}.\widehat{\mathbf{s}} = \cos\psi\cos\theta - \sin\psi\sin\theta\sin\varphi$, where $\cos\psi = \widehat{\mathbf{k}}.\widehat{\mathbf{p}}$. Also, we have $\gamma.\widehat{\mathbf{s}} = \gamma'^1 \sin\theta\cos\varphi + \gamma'^2(\sin\psi\cos\theta + \cos\psi\sin\theta\sin\varphi) + \gamma'^3(\cos\psi\cos\theta - \sin\psi\sin\theta\sin\varphi)$, where $\{\gamma'^i\}$ are the three spatially rotated Dirac matrices written in a reference frame where $\widehat{\mathbf{p}}$ is the principle axis and $\widehat{\mathbf{k}}$ in the (y,z)-plane. They are fixed in the integration over Ω_s. Performing all the (anti)commutations, we have:

$$
I_1 = \frac{8}{T^2}\, T\widetilde{\sum_{k_0}} \int \frac{d^3k}{(2\pi)^3} \int \frac{d\Omega_s}{4\pi} \frac{\gamma^0(1-\cos\theta) + i\gamma.\widehat{\mathbf{k}} - i\gamma.\widehat{\mathbf{s}}}{PS\,KS} \Delta_-\left(K\right)\, {}^*\Delta_l\left(Q\right) . \qquad (20)
$$

In order to be able to perform with ease the above solid-angle integral, we use the expansion:

$$
\frac{1}{PS} = \frac{1}{ip_0}\left[1 - \frac{\mathbf{p}.\widehat{\mathbf{s}}}{ip_0} - \frac{\mathbf{p}.\widehat{\mathbf{s}}^2}{p_0^2} + \dots \right] . \qquad (21)
$$

This expansion is valid in the region $p < |ip_0|$, a condition always satisfied before analytic continuation and after. Before because $p_0 = (2n+1)\,\pi T$ and $p \sim g^2 T$. After because for very soft momenta, $ip_0 = m_f + O(p/m_f) \sim gT$, see (5). The solid-angle integral in (20) then reads:

$$
\begin{aligned}
&\frac{1}{ip_0} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta\, \sin\theta \frac{\gamma^0(1-\cos\theta) + i\gamma.\widehat{\mathbf{k}} - i\gamma.\widehat{\mathbf{s}}}{ik_0 + k\cos\theta} \\
&\times \left[1 - \frac{p}{ip_0}(\cos\psi\cos\theta - \sin\psi\sin\theta\sin\varphi) - \frac{p^2}{p_0^2}(\cos\psi\cos\theta - \sin\psi\sin\theta\sin\varphi)^2 + \dots \right] .
\end{aligned}
$$

The angular integrations are now straightforward and we obtain:

$$
\begin{aligned}
I_1 = \frac{8}{T^2}\, T\widetilde{\sum_{k_0}} \int \frac{d^3k}{(2\pi)^3} \frac{1}{ip_0 k} \Bigg[\gamma^0 \Bigg[-1 + \left(1 + \frac{ik_0}{k}\right) Q_{0k} - \frac{px}{ip_0} \left(1 + \frac{ik_0}{k}\right)\left(1 - \frac{ik_0}{k} Q_{0k}\right) \\
+ \frac{p^2}{p_0^2} \Bigg[x^2 \left(\frac{1}{3} + \frac{ik_0}{k} - \frac{k_0^2}{k^2} + \frac{k_0^2}{k^2}\left(1 + \frac{ik_0}{k}\right) Q_{0k} \right) - \frac{1}{2}\left(1 - x^2\right)\left(-\frac{2}{3} + \frac{ik_0}{k} - \frac{k_0^2}{k^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\left(1+\frac{ik_0}{k}\right)\left(1+\frac{k_0^2}{k^2}\right)Q_{0k}\right)\Bigg]\Bigg] + i\gamma.\widehat{\mathbf{k}}\left[Q_{0k} - \frac{px}{ip_0}\left(1-\frac{ik_0}{k}Q_{0k}\right) + \frac{p^2}{p_0^2}\left[x^2\frac{ik_0}{k}\left(1-\frac{ik_0}{k}Q_{0k}\right)\right.\right. \\
&-\frac{1}{2}(1-x^2)\left(\frac{ik_0}{k}+\left(1+\frac{k_0^2}{k^2}\right)Q_{0k}\right)\Bigg]\Bigg] - i\gamma'^2\sin\psi\left[1-\frac{ik_0}{k}+\frac{px}{2ip_0}\left(3\frac{ik_0}{k}+\left(1+3\frac{k_0^2}{k^2}\right)Q_{0k}\right)\right. \\
&+\frac{p^2}{p_0^2}\left[x^2\left(\frac{1}{3}+2\frac{k_0^2}{k^2}-\frac{ik_0}{k}\left(1+2\frac{k_0^2}{k^2}\right)Q_{0k}\right) - \frac{1}{2}(1-x^2)\left(\frac{2}{3}+\frac{k_0^2}{k^2}-\frac{ik_0}{k}\left(1+\frac{k_0^2}{k^2}\right)Q_{0k}\right)\right]\Bigg] \\
&-i\gamma'^3\left[x\left(1-\frac{ik_0}{k}Q_{0k}\right)+\frac{p}{ip_0}\left[x^2\left(\frac{ik_0}{k}+\frac{k_0^2}{k^2}Q_{0k}\right)-\frac{1}{2}(1-x^2)\left(\frac{ik_0}{k}+\left(1+\frac{k_0^2}{k^2}\right)Q_{0k}\right)\right]\right. \\
&+\frac{p^2}{p_0^2}\left[x^3\left(\frac{ik_0}{3k}-\frac{ik_0^3}{k^3}-\frac{k_0^4}{k^4}Q_{0k}\right)-\frac{3}{2}x(1-x^2)\left(\frac{2}{3}+\frac{k_0^2}{k^2}-\frac{ik_0}{k}\left(1+\frac{k_0^2}{k^2}\right)Q_{0k}\right)\right]\Bigg]+\ldots\Bigg] \\
&\times\Delta_-\left(K\right)\,{}^*\Delta_l\left(Q\right)\,, \qquad (22)
\end{aligned}
$$

where $x=\cos\psi$ and Q_{0k} stands for $Q_0(ik_0/k)$.

The next step is to perform the integrals over the solid angle of $\widehat{\mathbf{k}}$ in a reference frame where $\widehat{\mathbf{p}}$ is the principle axis. For this, it is most useful to develop all functions of $q=|\mathbf{p}-\mathbf{k}|$ around k for (very) small p. The validity of these expansions is discussed in the previous section. In particular, here we need:

$$
{}^*\Delta_{l,t}\left(q_0,q\right)=\left[1-px\,\partial_k+\frac{p^2}{2}\left(\frac{1-x^2}{k}\partial_k+x^2\partial_k^2\right)+\ldots\right]{}^*\Delta_{l,t}\left(q_0,k\right)\,. \qquad (23)
$$

The solid angle of $\widehat{\mathbf{k}}$ is $\Omega_k=(\psi,\alpha)$ and we have the relation:

$$
\gamma'^1=\gamma^1\cos\alpha-\gamma^2\sin\alpha;\quad \gamma'^2=\gamma^1\sin\alpha+\gamma^2\cos\alpha;\quad \gamma'^3=\gamma^3\,, \qquad (24)
$$

where the $\{\gamma^i\}$ are the (fixed) spatial Dirac matrices. Using (23), the integrations over ψ and α become straightforward. We obtain:

$$
\begin{aligned}
I_1 = &\frac{4}{\pi^2T^2}\,T\widetilde{\sum_{k_0}}\int_\eta^{+\infty}dk\frac{k}{ip_0}\Delta_{-K}\left[\gamma^0\left[-1+\left(1+\frac{ik_0}{k}\right)Q_{0k}+\frac{p^2}{3}\left[\frac{1}{p_0^2}\left(1-\left(1+\frac{ik_0}{k}\right)Q_{0k}\right)\right.\right.\right. \\
&+\frac{1}{ip_0}\left(1+\frac{ik_0}{k}\right)\left(1-\frac{ik_0}{k}Q_{0k}\right)\partial_k-\left(1-\left(1+\frac{ik_0}{k}\right)Q_{0k}\right)\left(\frac{1}{k}\partial_k+\frac{1}{2}\partial_k^2\right)\Bigg]\Bigg] \\
&+\frac{1}{3}i\gamma^3p\left(1-\left(1+\frac{ik_0}{k}\right)Q_{0k}\right)\left(-\frac{1}{ip_0}+\partial_k\right)+\ldots\Bigg]\,{}^*\Delta_l\left(q_0,k\right)\,, \qquad (25)
\end{aligned}
$$

where Δ_{-K} stands for $\Delta_-(K)$. Note the introduction of the infrared cut-off η. Note also that terms proportional to p do not vanish, contrary to what happens for gluons [33,34].

The next step for I_1 is to perform the Matsubara sum. This will be done after we get for the first term I_0 in ${}^*\Sigma_{-l}(P)$ (the one that involves no angular integrals) and the third term I_2 (the one that involves two such integrals) expressions similar to (25). As for I_0, the calculation is simpler: only an integral over Ω_k using the expansion (23) is needed. We get:

$$I_0 = \frac{2}{3\pi^2 T^2}\, T\widetilde{\sum_{k_0}} \int_\eta^{+\infty} dk\, k^2 \Delta_{-K} \left[\gamma^0 \left[3 + p^2 \left(\frac{1}{k}\partial_k + \frac{1}{2}\partial_k^2\right)\right] - i\gamma^3 p \partial_k + \ldots\right] {}^*\Delta_l\left(q_0, k\right)\,. \quad (26)$$

As for I_2, more work is needed. But we are fortunate here since $\widehat{\mathbf{s}}_1$ is not 'coupled' to $\widehat{\mathbf{s}}_2$ so that each solid-angle integral can be performed independently from the other. Each integral is thus performed along the lines shown for I_1, and so, there is no need to re-display the steps. After the two integrations are done, we multiply the two results, keeping terms to order p^2 only and taking care of the Dirac algebra. We obtain:

$$\begin{aligned} I_2 = \frac{2}{\pi^2 T^2} T\widetilde{\sum_{k_0}} \int_\eta^{+\infty} \frac{dk}{p_0^2} \Delta_{-K} \Bigg[-\gamma^0 a_-^2 + i\gamma^3 \frac{p}{3} a_-^2 \left(-\frac{2}{ip_0} + \partial_k\right) + \gamma^0 \frac{p^2}{3} \left[\frac{1}{2p_0^2}\left(3 - \left(2 - 6\frac{ik_0}{k}\right) a_- \right.\right. \\ \left.+ \left(5 - 2\frac{ik_0}{k} - 3\frac{k_0^2}{k^2}\right) a_-^2\right) + \frac{2}{ip_0} a_- \left(1 + \frac{ik_0}{k} a_-\right)\partial_k - a_- \left(\frac{1}{k}\partial_k + \frac{1}{2}\partial_k^2\right)\Bigg] + \ldots \Bigg] \\ \times {}^*\Delta_l\left(q_0, k\right), \end{aligned} \quad (27)$$

where we have denoted for short $a_\varepsilon = 1 + \varepsilon\left(1 - \varepsilon\frac{ik_0}{k}\right) Q_{0k}$, $\varepsilon = \pm$. We can now put together I_0, I_1 and I_2 to get a first expression for ${}^*\Sigma_{-l}(P)$. Since ${}^*\Sigma_{+l}(P)$ is calculated in the same way and the only differences are mere signs, it is more economical to write the result for both terms in one single expression. We find:

$$\begin{aligned} {}^*\Sigma_{\varepsilon l}(P) = \frac{2}{\pi^2 T^2}\, T\widetilde{\sum_{k_0}} \int_\eta^{+\infty} dk\, k^2 \Delta_{\varepsilon K} \Bigg[\gamma^0 \left(1 + \frac{2\varepsilon}{ip_0 k} a_\varepsilon - \frac{1}{p_0^2 k^2} a_\varepsilon^2\right) \\ + i\gamma^3 \frac{p}{3}\left[\varepsilon\partial_k + \frac{2}{ip_0 k} a_\varepsilon \left(\frac{\varepsilon}{ip_0} + \partial_k\right) - \frac{\varepsilon}{p_0^2 k^2} a_\varepsilon^2 \left(\frac{2\varepsilon}{ip_0} + \partial_k\right)\right] \\ + \gamma^0 \frac{p^2}{3}\Bigg[\left(\frac{1}{k}\partial_k + \frac{1}{2}\partial_k^2\right) + \frac{2}{ip_0 k}\left(-\frac{\varepsilon}{p_0^2} a_\varepsilon + \frac{1}{ip_0}\left(1 - \varepsilon\frac{ik_0}{k} a_\varepsilon\right)\partial_k + \varepsilon a_\varepsilon \left(\frac{1}{k}\partial_k + \frac{1}{2}\partial_k^2\right)\right) \\ + \frac{1}{p_0^2 k^2}\Bigg(\frac{1}{2p_0^2}\left(3 - 2\left(1 + 3\varepsilon\frac{ik_0}{k}\right) a_\varepsilon + \left(5 + 2\varepsilon\frac{ik_0}{k} - 3\frac{k_0^2}{k^2}\right) a_\varepsilon^2\right) - \frac{2\varepsilon}{ip_0} a_\varepsilon \left(1 - \varepsilon\frac{ik_0}{k} a_\varepsilon\right)\partial_k \\ - a_\varepsilon^2 \left(\frac{1}{k}\partial_k + \frac{1}{2}\partial_k^2\right)\Bigg)\Bigg] + \ldots\Bigg]\, {}^*\Delta_l\left(q_0, k\right)\,. \end{aligned} \quad (28)$$

Now we are ready to perform the Matsubara sum over fermionic k_0. We will need the spectral decomposition of $\Delta_\varepsilon\left(k_0, k\right)$, ${}^*\Delta_l\left(q_0, k\right)$ and $Q_0(ik_0/k)$. They are worked out in [43,15] and are given by:

$$\begin{aligned} \Delta_\varepsilon(k_0, k) &= \int_0^{1/T} d\tau\, e^{ik_0\tau} \int_{-\infty}^{+\infty} d\omega\, \rho_\varepsilon(\omega, k)\left(1 - \tilde{n}(\omega)\right) e^{-\omega\tau}\,; \\ \Delta_{t,l}(k_0, k) &= \int_0^{1/T} d\tau\, e^{ik_0\tau} \int_{-\infty}^{+\infty} d\omega\, \rho_{t,l}(\omega, k)\left(1 + n(\omega)\right) e^{-\omega\tau}\,; \\ Q_0(ik_0/k) &= \int_0^{1/T} d\tau\, e^{ik_0\tau} \int_{-\infty}^{+\infty} d\omega\, \rho_0(\omega, k)\left(1 - \tilde{n}(\omega)\right) e^{-\omega\tau}\,. \end{aligned} \quad (29)$$

$n(\omega)$ ($\tilde{n}(\omega)$) is the Bose-Einstein (Fermi-Dirac) distribution and the rho's are the spectral densities. Before replacing these above quantities, it is first necessary to rearrange terms in

(28) in a such a way that products of only two such functions appear. The reason behind is to ensure the appearance of only one energy denominator just before the extraction of the imaginary part, see below. For this purpose, we use the following easy-to-check relations:

$$\begin{aligned} a_\varepsilon \Delta_\varepsilon &= -\varepsilon k \left[1 - \left(ik_0 - \varepsilon k\right) \Delta_\varepsilon\right] ; \\ a_\varepsilon^2 \Delta_\varepsilon &= -\varepsilon k \left[a_\varepsilon + \varepsilon k \left(ik_0 - \varepsilon k\right) \left[1 - \left(ik_0 - \varepsilon k\right) \Delta_\varepsilon\right]\right] . \end{aligned} \tag{30}$$

After rearrangements, many terms happen to be real. Dropping these will yield:

$$\begin{aligned} \mathrm{Im}\ {}^*\Sigma_{\varepsilon l}(P) &= \frac{2}{\pi^2 T^2} \mathrm{Im}\ T \widetilde{\sum_{k_0}} \int_\eta^{+\infty} dk\, k^2 \left[\gamma^0 \left(\left(1 + \frac{ik_0 - \varepsilon k}{ip_0}\right)^2 \Delta_\varepsilon + \frac{\varepsilon}{p_0^2 k} a'_\varepsilon\right)\right. \\ & -i\gamma^3 \frac{p}{3} \left[2 \frac{ik_0 - \varepsilon k}{p_0^2} \left(1 + \frac{ik_0 - \varepsilon k}{ip_0}\right) \Delta_\varepsilon - \varepsilon \left(1 + \frac{ik_0 - \varepsilon k}{ip_0}\right)^2 \Delta_\varepsilon \partial_k - \frac{1}{p_0^2 k} a'_\varepsilon \left(\frac{2\varepsilon}{ip_0} + \partial_k\right)\right] \\ & +\gamma^0 \frac{p^2}{3} \left[\left(\frac{3}{2p_0^4 k^2} - 2\frac{ik_0 - \varepsilon k}{ip_0^3} + \frac{\left(ik_0 - \varepsilon k\right)^2}{2p_0^4} \left(5 + 2\varepsilon \frac{ik_0}{k} - 3\frac{k_0^2}{k^2}\right)\right.\right. \\ & \left. - \varepsilon \frac{ik_0 - \varepsilon k}{p_0^4 k} \left(1 + 3\varepsilon \frac{ik_0}{k}\right)\right) \Delta_\varepsilon + \left(\frac{1}{k} - \frac{2}{p_0^2 k} + 2\frac{ik_0 \left(ik_0 - \varepsilon k\right)}{p_0^2 k} + 2\frac{ik_0 - \varepsilon k}{ip_0 k}\right. \\ & \left. + 2\frac{ik_0 \left(ik_0 - \varepsilon k\right)^2}{ip_0^3 k} - 2\frac{ik_0 - \varepsilon k}{ip_0^3 k} - \frac{\left(ik_0 - \varepsilon k\right)^2}{p_0^2 k}\right) \Delta_\varepsilon \partial_k + \frac{1}{2} \left(1 + \frac{ik_0 - \varepsilon k}{ip_0}\right)^2 \Delta_\varepsilon \partial_k^2 \\ & \left.\left. - \frac{\varepsilon}{2p_0^4 k} \left(5 + 2\varepsilon \frac{ik_0}{k} - 3\frac{k_0^2}{k^2}\right) a'_\varepsilon - \varepsilon \left(2\frac{ik_0}{ip_0^3 k^2} - \frac{1}{p_0^2 k^2}\right) a'_\varepsilon \partial_k + \frac{\varepsilon}{2p_0^2 k} a'_\varepsilon \partial_k^2\right] + \ldots\right] {}^*\Delta_l \left(q_0, k\right) . \end{aligned} \tag{31}$$

Here $a'_\varepsilon = a_\varepsilon - 1 = \varepsilon \left(1 - \varepsilon \frac{ik_0}{k}\right) Q_{0k}$. Since ik_0 appears in (31) only in the numerator of fractions, we can sum over it using the spectral decompositions (29). At each time, we are left with two frequency integrals together with the one over k. Now we are allowed to take the real-energy analytic continuation $ip_0 \to \omega_\pm(p) + i0^+$. But just before, every $e^{\frac{ip_0}{T}}$ has to be replaced with -1 except in the energy denominators which occur only once in each term, thanks to the rearrangements we made using (30). The extraction of the imaginary part becomes straightforward if we use the relation $1/\left(x + i0^+\right) = \mathrm{Pr}\left(1/x\right) - i\pi\delta(x)$. We obtain the following expression:

$$\begin{aligned} \mathrm{Im}\ {}^*\Sigma_{\varepsilon l}(P) &= \frac{2}{\pi T} \int_\eta^{+\infty} dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'} \delta\left(\omega_\pm - \omega - \omega'\right) \Big[\gamma^0 \Big(k^2 \left(1 + \omega - \varepsilon k\right)^2 \rho_\varepsilon \\ & - \left(k - \varepsilon\omega\right) \rho_0\Big) + \frac{p}{3} \Big[\mp 2\gamma^0 \left(\omega - \varepsilon k\right) \Big(k^2 \left(1 + \omega - \varepsilon k\right) \rho_\varepsilon + \varepsilon \rho_0\Big) \\ & + i\gamma^3 \Big(2k^2 \left(\omega - \varepsilon k\right) \left(1 + \omega - \varepsilon k\right) \rho_\varepsilon + \varepsilon k^2 \left(1 + \omega - \varepsilon k\right)^2 \rho_\varepsilon \partial_k + \left(\omega - \varepsilon k\right) \rho_0 \left(2\varepsilon + \partial_k\right)\Big)\Big] \\ & + \frac{p^2}{3} \Big[\gamma^0 \Big[\Big(\frac{3}{2} - \varepsilon \left(\omega - \varepsilon k\right) \left(k + 3\varepsilon\omega\right) + \frac{2}{3} k^2 \left(\omega - \varepsilon k\right) + \frac{1}{2} \left(\omega - \varepsilon k\right)^2 \Big(3k^2 + 2\varepsilon\omega k + 3\omega^2\Big)\Big) \rho_\varepsilon \\ & + k \Big(3 + 4\left(\omega - \varepsilon k\right) + k^2 - \omega^2 - 2\omega \left(\omega - \varepsilon k\right)^2\Big) \rho_\varepsilon \partial_k + \frac{k^2}{2} \left(1 + \omega - \varepsilon k\right)^2 \rho_\varepsilon \partial_k^2 \\ & + \frac{\varepsilon}{2k^2} \left(\omega - \varepsilon k\right) \Big(3k^2 + 2\varepsilon\omega k + 3\omega^2\Big) \rho_0 - \frac{\varepsilon}{k} \left(\omega - \varepsilon k\right) \left(2\omega - 1\right) \rho_0 \partial_k + \frac{\varepsilon}{2} \left(\omega - \varepsilon k\right) \rho_0 \partial_k^2\Big] \end{aligned}$$

$$\mp\frac{1}{3}i\gamma^3\left[2k^2\left(\omega-\varepsilon k\right)\left(2+3\omega-3\varepsilon k\right)\rho_\varepsilon+2\varepsilon k^2\left(1+\omega-\varepsilon k\right)\left(\omega-\varepsilon k\right)\rho_\varepsilon\partial_k\right.$$
$$\left.+\ 2\left(\omega-\varepsilon k\right)\rho_0\left(3\varepsilon+\partial_k\right)\right]]+\ldots]\,\rho_l'\,. \tag{32}$$

Recall that we have set $m_f=1$. The notation is as follows: $\rho_{\varepsilon,0}=\rho_{\varepsilon,0}(\omega,k)$; $\rho_l'=\rho_l(\omega',k)$. In the above expression, we have used $\tilde{n}(\omega)\simeq\frac{1}{2}$ and $n(\omega)\simeq T/\omega$. This is because only soft values of ω and ω' are to contribute. The resulting integrals are to be performed numerically, but after the extraction of potential infrared divergences.

It remains to calculate the two other contributions to $\mathrm{Im}^*\,\Sigma_1$, those coming from transverse gluons in (18). The final result is the following:

$$\mathrm{Im}\ {}^*\Sigma_{\varepsilon t}(P)=\frac{2}{\pi T}\int_\eta^{+\infty}dk\ \int_{-\infty}^{+\infty}d\omega\int_{-\infty}^{+\infty}\frac{d\omega'}{\omega'}\delta\left(\omega_\pm-\omega-\omega'\right)\Big[\gamma^0\Big[\Big(-\frac{1}{2}(2k+\varepsilon)^2$$
$$-\frac{1}{2}\left(k^2-\omega^2\right)^2-\varepsilon\left(2k+\varepsilon\right)\left(k^2-\omega^2\right)\Big)\rho_\varepsilon+\frac{1}{2k^2}\left(\varepsilon\omega+k\right)\left(k^2-\omega^2\right)\rho_0\Big]$$
$$+\frac{p}{3}\Big[\mp\gamma^0\Big[-\Big(\left(k^2-\omega^2\right)^2+2\left(1+\varepsilon k\right)\left(k^2-\omega^2\right)+2\varepsilon k+1\Big)\rho_\varepsilon$$
$$+\frac{1}{k^2}\left(k+\varepsilon\omega\right)\left(k^2-\omega^2\right)\rho_0\Big]+i\gamma^3\Big[\Big(-\left(k^2-\omega^2\right)^2-\frac{2\varepsilon}{k}\left(k^2-\omega^2\right)\left(k^2+\varepsilon\omega k-\omega^2\right)$$
$$+4\omega(\omega-\varepsilon k)+4\varepsilon\frac{\omega^2}{k}-2\omega-3-\frac{2\varepsilon}{k}\Big)\rho_\varepsilon+\varepsilon\Big(\frac{1}{2}\left(k^2-\omega^2\right)^2+2\varepsilon k\left(k^2-\omega^2\right)+3k^2-\omega^2$$
$$+\ 2\varepsilon k+\frac{1}{2}\Big)\rho_\varepsilon\partial_k+\Big(2\frac{\omega}{k^3}+\frac{1}{k^2}\left(k+\varepsilon\omega\right)\Big)\left(k^2-\omega^2\right)\rho_0-\frac{\varepsilon}{2k^2}\left(k+\varepsilon\omega\right)^2\left(k-\varepsilon\omega\right)\rho_0\partial_k\Big]\Big]$$
$$+\frac{p^2}{3}\Big[\gamma^0\Big[\Big[-k^2\left(\omega-\varepsilon k\right)^2\Big(1+\varepsilon\frac{\omega}{k}\Big)\Big(1+\varepsilon\frac{\omega^3}{k^3}\Big)+k\left(\omega-\varepsilon k\right)\Big(\frac{2}{3}k+\frac{11\varepsilon}{3}\omega-\frac{\omega^2}{k}-3\varepsilon\frac{\omega^3}{k^2}+\frac{\omega^4}{k^3}\Big)$$
$$-\frac{k}{2}\left(\omega-\varepsilon k\right)\Big(\frac{7\varepsilon}{3}-\frac{13\omega}{3k}-\frac{47\varepsilon}{3k^2}\omega^2-\frac{\omega^3}{k^3}\Big)-k\Big(\frac{4\varepsilon}{3}-\frac{10\omega}{3k}+\frac{2\varepsilon}{3k^2}\omega^2+\frac{2\omega^3}{k^3}\Big)-\frac{4}{9}-\frac{25\varepsilon}{3k}\omega+2\frac{\omega^2}{k^2}$$
$$+\frac{14\varepsilon}{3k}+\frac{\omega}{k^2}-\frac{3}{2k^2}\Big)\rho_\varepsilon+\Big(\frac{\omega}{k}\left(k^2-\omega^2\right)^2-\frac{1}{2k}\left(k^2-\omega^2\right)\left(k^2-\omega^2-4\varepsilon k\omega\right)$$
$$+\frac{1}{k}\left(\omega-\varepsilon k\right)\Big(\frac{4}{3}k^2-\frac{2\varepsilon}{3}k\omega-2\omega^2\Big)-\frac{5}{3}k+2\varepsilon\omega+\frac{\omega^2}{k}-\frac{4\varepsilon}{3}+\frac{\omega}{k}-\frac{1}{2k}\Big)\rho_\varepsilon\partial_k$$
$$+\frac{1}{4}\Big(-\left(\omega^2-k^2\right)^2+4\varepsilon k\left(\omega^2-k^2\right)-6k^2-4\varepsilon k+2\omega^2-1\Big)\rho_\varepsilon\partial_k^2-\varepsilon\Big(\Big(1+\varepsilon\frac{\omega}{k}\Big)\Big(1+\varepsilon\frac{\omega^3}{k^3}\Big)$$
$$+\frac{\omega}{k^2}\Big(3+2\varepsilon\frac{\omega}{k}-\frac{\omega^2}{k^2}\Big)-\frac{1}{2k^2}\Big(1+2\varepsilon\frac{\omega}{k}-3\frac{\omega^2}{k^2}\Big)\Big)\left(\omega-\varepsilon k\right)\rho_0-\varepsilon\Big(\frac{1}{2k^3}-\frac{\omega}{k^3}\Big)$$
$$\times\left(\omega^2-k^2\right)\left(\omega+\varepsilon k\right)\rho_0\partial_k+\frac{1}{4k^2}\left(k+\varepsilon\omega\right)^2\left(k-\varepsilon\omega\right)\rho_0\partial_k^2\Big]$$
$$\mp\frac{1}{3}i\gamma^3[(-3\left(k^2-\omega^2\right)^2-\frac{4\varepsilon}{k}\left(k^2-\omega^2\right)\left(k^2+\varepsilon k\omega-\omega^2\right)+4\left(\omega-\varepsilon k\right)\left(2\omega+\varepsilon k\right)$$
$$-4\Big(2\varepsilon k+\omega-2\varepsilon\frac{\omega^2}{k}\Big)-5-\frac{4\varepsilon}{k}\Big)\rho_\varepsilon+\Big(\varepsilon\left(k^2-\omega^2\right)^2+2\left(k+\varepsilon\right)\left(k^2-\omega^2\right)+2k+\varepsilon\Big)\rho_\varepsilon\partial_k$$
$$+\ \Big(\frac{3}{k^2}\left(k+\varepsilon\omega\right)+4\frac{\omega}{k^3}\Big)\left(k^2-\omega^2\right)\rho_0+\frac{1}{k^2}\left(\omega-\varepsilon k\right)\left(k+\varepsilon\omega\right)^2\rho_0\partial_k\Big]\Big]+\ldots\Big]\,\rho_t'\,. \tag{33}$$

This expression is quite long because these two terms are more involved. However, there are not new steps worth discussing in detail.

B. Calculation of $\mathrm{Im}\,{}^*\Sigma_2\,(P)$

Now we turn to calculating the imaginary part of ${}^*\Sigma_2\,(P)$ which can be written from (12) and the structure of the gluon propagator in the strict Coulomb gauge as:

$$ {}^*\Sigma_2\,(P) = -\frac{8}{T^2}\,T\sum_{k_0}\int\frac{d^3k}{(2\pi)^3}\int\frac{d\Omega_s}{4\pi}\,\frac{i\not{S}}{KS\,PS\,QS}\left[{}^*\Delta_l(K)-\left(1-\widehat{\mathbf{k}}.\widehat{\mathbf{s}}^2\right){}^*\Delta_t(K)\right]. \tag{34} $$

The steps to carry are similar to the ones we used for the previous contribution. But here we need the additional expansion:

$$ \frac{1}{QS} = \frac{1}{iq_0-\widehat{\mathbf{k}}.\widehat{\mathbf{s}}}\left[1-\frac{\widehat{\mathbf{p}}.\widehat{\mathbf{s}}}{iq_0-\widehat{\mathbf{k}}.\widehat{\mathbf{s}}}-\frac{\widehat{\mathbf{p}}.\widehat{\mathbf{s}}^2}{\left(iq_0-\widehat{\mathbf{k}}.\widehat{\mathbf{s}}\right)^2}+\ldots\right]. \tag{35} $$

The angular integrals over the solid angles Ω_s and then Ω_k are done as usual. We obtain for the longitudinal gluon:

$$ \begin{aligned} {}^*\Sigma_{2l}\,(P) \;=& -\frac{4}{\pi^2T^2}\,T\sum_{k_0}\int_\eta^{+\infty}k\,dk\left[\gamma^0\frac{1}{p_0^2}Q'_{0k}+i\gamma^3\frac{p}{3}\left[\frac{2}{ip_0^3}Q'_{0k}-\frac{k}{p_0^2\,(q_0^2+k^2)}\right]\right.\\ &\left.+\,\gamma^0\frac{p^2}{p_0^2}\left[-\frac{1}{p_0^2}Q'_{0k}-\frac{2k}{3ip_0\,(q_0^2+k^2)}+\frac{iq_0k}{3\,(q_0^2+k^2)^2}\right]+\ldots\right]{}^*\Delta_l(k_0,k)\,, \end{aligned} \tag{36} $$

and for the transverse one:

$$ \begin{aligned} {}^*\Sigma_{2t}\,(P) \;=& -\frac{4}{\pi^2T^2}\,T\sum_{k_0}\int_\eta^{+\infty}\frac{dk}{k}\left[-\gamma^0\frac{1}{p_0^2}\left(q_0^2+k^2\right)Q'_{0k}-i\gamma^3\frac{2p}{3p_0^2}\left(\frac{1}{ip_0}\left(q_0^2+k^2\right)+iq_0\right)Q'_{0k}\right.\\ &\left.+\,\gamma^0\frac{p^2}{p_0^2}\left[\left(\frac{1}{p_0^2}\left(q_0^2+k^2\right)-\frac{4iq_0}{3ip_0}+\frac{1}{3}\right)Q'_{0k}+\frac{iq_0k}{3\,(q_0^2+k^2)}\right]+\ldots\right]{}^*\Delta_t(k_0,k)\,, \end{aligned} \tag{37} $$

where Q'_{0k}stands for $Q_0\left(\frac{iq_0}{k}\right)$. The sum over bosonic k_0 is now readily done if we add to the spectral representations (29) those of $1/\left(q_0^2+k^2\right)$ and $1/\left(q_0^2+k^2\right)^2$. We have:

$$ \begin{aligned} \frac{1}{(q_0^2+k^2)} &= \int_0^{1/T}d\tau\,e^{iq_0\tau}\int_{-\infty}^{+\infty}d\omega\,\epsilon(\omega)\delta(\omega^2-k^2)\,(1-\tilde{n}(\omega))\,e^{-\omega\tau}\,;\\ \frac{1}{(q_0^2+k^2)^2} &= \int_0^{1/T}d\tau\,e^{iq_0\tau}\int_{-\infty}^{+\infty}d\omega\,\epsilon(\omega)\,\delta^{(1)}(\omega^2-k^2)\,(1-\tilde{n}(\omega))\,e^{-\omega\tau}\,, \end{aligned} \tag{38} $$

with q_0 fermionic. $\epsilon(\omega)$ is the sign function and $\delta^{(1)}(\omega^2-k^2)$ stands for $\partial_{\omega^2}\delta\left(\omega^2-k^2\right)$. The extraction of the imaginary part is straightforward. We obtain for the longitudinal contribution:

$$\text{Im }{}^*\Sigma_{2l}(P) = -\frac{4}{\pi T}\int_\eta^{+\infty} dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'}\delta(\omega_\pm - \omega - \omega')\Big[-\gamma^0 k\rho_0 + \frac{p}{3}\Big[\pm 2\gamma^0 k\rho_0$$
$$+ i\gamma^3\left(-2k\rho_0 + k^2\epsilon(\omega)\delta\left(\omega^2 - k^2\right)\right)\Big] + p^2\Big[\gamma^0\Big(-\frac{2k}{3}\rho_0 + \frac{2k^2}{3}\epsilon(\omega)\delta(\omega^2 - k^2)$$
$$- \frac{k^2}{3}\omega\epsilon(\omega)\,\delta^{(1)}(\omega^2 - k^2)\Big) \mp \frac{1}{3}i\gamma^3\left(-2k\rho_0 + \frac{2k^2}{3}\epsilon(\omega)\delta\left(\omega^2 - k^2\right)\right)\Big] + \ldots\Big]\,\rho_l', \quad (39)$$

and for the transverse one:

$$\text{Im }{}^*\Sigma_{2t}(P) = -\frac{4}{\pi T}\int_\eta^{+\infty} dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'}\delta(\omega_\pm - \omega - \omega')\Big[\gamma^0\frac{1}{k}\left(k^2 - \omega^2\right)\rho_0$$
$$+\frac{p}{3}\Big[\mp\gamma^0\frac{2}{k}\left(k^2 - \omega^2\right)\rho_0 + i\gamma^3\frac{2}{k}\left(k^2 + \omega - \omega^2\right)\rho_0\Big] + p^2\Big[\gamma^0\Big(\frac{2}{3k}\left(k^2 - \frac{1}{2} + 2\omega - \omega^2\right)\rho_0$$
$$-\frac{\omega}{3}\epsilon(\omega)\delta\left(\omega^2 - k^2\right)\Big) \mp i\gamma^3\frac{2}{3k}\left(k^2 + \frac{2}{3}\omega - \omega^2\right)\rho_0\Big] + \ldots\Big]\,\rho_t'. \quad (40)$$

The final result we aim at is the sum of the six terms:

$$\text{Im }{}^*\Sigma(P) = \sum_{\varepsilon=\pm,\, i=l,t} \text{Im }{}^*\Sigma_{\varepsilon i}(P) + \sum_{i=l,t} \text{Im }{}^*\Sigma_{2i}(P)\,, \quad (41)$$

where the different contributions are given in (32), (33), (39) and (40). To get the damping rates $\gamma_\pm(p)$, we use eq (9) where ${}^*f_\pm = {}^*D_0 \mp {}^*D_s$ and ${}^*\Sigma = \gamma^0\,{}^*D_0 + i\gamma.\widehat{\mathbf{p}}\;{}^*D_s$. There are few more steps though. Indeed, note that the energy $\omega_\pm(p)$ appearing in $\delta(\omega_\pm - \omega - \omega')$ is a function of p, given in (5) for small p. This means that for the terms in p^2, the energy $\omega_\pm(p)$ can be replaced by one (in units of m_f) since we look for the damping rates up to order p^2, but for the terms of order p, we have to expand $\delta(\omega_\pm - \omega - \omega')$ to order p and for the terms of order zero to order p^2. A subsequent rearrangement is necessary.

V. RESULTS AND CONCLUSION

The damping rates are given in (9). We find:

$$\gamma_\pm(p) = -\frac{g^2 C_f T}{8\pi}\left[a_0 \pm \frac{p}{3}a_1 + \frac{p^2}{9}a_2 + \ldots\right], \quad (42)$$

where the coefficients a_i are given by the expressions:

$$a_0 = \int_\eta^\infty dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'} f_0(\omega,\omega';k)\,\delta\,;$$
$$a_1 = \int_\eta^\infty dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'}\left[\,f_1(\omega,\omega';k) - f_0(\omega,\omega';k)\;\partial_\omega\right]\delta,$$
$$a_2 = \int_\eta^\infty dk \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} \frac{d\omega'}{\omega'}\Big[f_2(\omega,\omega';k) - f_1(\omega,\omega';k)\,\partial_\omega + f_0(\omega,\omega';k)\left[-3\,\partial_\omega + \partial_\omega^2\right]\Big]\,\delta, \quad (43)$$

with $\delta = \delta\left(1 - \omega - \omega'\right)$. The three functions $f_i\left(\omega, \omega'; k\right)$ are given by the following expressions:

$$\begin{aligned} f_0\left(\omega, \omega'; k\right) = & \sum_{\varepsilon=\pm}\left[-k^2\left(1 - \varepsilon k + \omega\right)^2 \rho_\varepsilon \rho_l' + \frac{1}{2}\left(1 + 2\varepsilon k + k^2 - \omega^2\right)^2 \rho_\varepsilon \rho_t'\right] \\ & + \frac{1}{k}\left(k^2 - \omega^2\right) \rho_0 \rho_t' \,. \end{aligned} \tag{44}$$

This expression is the one obtained in [38,39]. f_1 and f_2 are new. They read:

$$\begin{aligned} f_1\left(\omega, \omega'; k\right) = & \sum_{\varepsilon=\pm}\Big[2k^2\left(-1 + k^2 - 2\varepsilon k\omega + \omega^2\right)\rho_\varepsilon \rho_l' + \Big(-\frac{2\varepsilon}{k} - 3 + 2\varepsilon k + 4k^2 - k^4 \\ & - \left(2 + 4\varepsilon k + 2k^2\right)\omega + \left(\frac{4\varepsilon}{k} + 4 + 2\varepsilon k + 2k^2\right)\omega^2 + 2\omega^3 - \left(\frac{2\varepsilon}{k} + 1\right)\omega^4\Big]\rho_\varepsilon \rho_t' \\ & + \varepsilon k^2 (1 - \varepsilon k + \omega)^2 \rho_\varepsilon \partial_k \rho_l' + \left(\frac{\varepsilon}{2} + 2k + 3\varepsilon k^2 + 2k^3 + \frac{\varepsilon}{2}k^4 - \left(\varepsilon + 2k + \varepsilon k^2\right)\omega^2 + \frac{\varepsilon}{2}\omega^4\right)\rho_\varepsilon \partial_k \rho_t'\Big] \\ & - \frac{2}{k}\left(k^2 - \omega^2 + 2\frac{\omega^3}{k^2}\right)\rho_0 \rho_t' - 2k^2 \epsilon\left(\omega\right)\delta\left(\omega^2 - k^2\right)\rho_l' + \frac{\omega}{k^2}\left(\omega^2 - k^2\right)\rho_0 \partial_k \rho_t' + 2\omega \rho_0 \partial_k \rho_l' \,; \end{aligned} \tag{45}$$

and:

$$\begin{aligned} f_2\left(\omega, \omega'; k\right) = & \sum_{\varepsilon=\pm}\Big[\Big(-\frac{9}{2} - k^2 - 6\varepsilon k^3 - \frac{1}{2}k^4 - \left(6\varepsilon k - 6k^2 + 2\varepsilon k^3\right)\omega + \left(9 + k^2\right)\omega^2 + 6\varepsilon k\omega^3 \\ & - \frac{9}{2}\omega^4\Big)\rho_\varepsilon \rho_l' + \Big(\frac{9}{2k^2} - \frac{14\varepsilon}{k} - \frac{8}{3} + 4\varepsilon k - \frac{19}{2}k^2 - 6\varepsilon k^3 + k^4 + \left(\frac{-3}{k^2} + \frac{25\varepsilon}{k} - 10 + 6\varepsilon k + 9k^2 - 3\varepsilon k^3\right)\omega \\ & + \left(-\frac{6}{k^2} + \frac{2\varepsilon}{k} + 23 - 6\varepsilon k + k^2\right)\omega^2 + \left(\frac{6}{k^2} - \frac{22\varepsilon}{k} - 6 + 6\varepsilon k\right)\omega^3 + \left(-\frac{3}{2k^2} + \frac{12\varepsilon}{k} - 5\right)\omega^4 \\ & - \left(\frac{3}{k^2} + \frac{3\varepsilon}{k}\right)\omega^5 + \frac{3}{k^2}\omega^6\Big)\rho_\varepsilon \rho_t' - k\Big(9 - 14\varepsilon k + 5k^2 + \left(12 - 2\varepsilon k - 6k^2\right)\omega - \left(3 - 12\varepsilon k\right)\omega^2 \\ & - 6\omega^3\Big)\rho_\varepsilon \partial_k \rho_l' + \Big(\frac{3}{2k} + 4\varepsilon + 7k + 8\varepsilon k^2 + \frac{7}{2}k^3 - \left(\frac{3}{k} + 6\varepsilon + 6k + 6\varepsilon k^2 + 3k^3\right)\omega \\ & - \left(\frac{3}{k} + 4\varepsilon + 5k\right)\omega^2 + \left(\frac{6}{k} + 6\varepsilon + 6k\right)\omega^3 + \frac{3}{2k}\omega^4 - \frac{3}{k}\omega^5\Big)\rho_\varepsilon \partial_k \rho_t' - \frac{3}{2}k^2\left(1 - \varepsilon k + \omega\right)^2 \rho_\varepsilon \partial_k^2 \rho_l' \\ & + \left(\frac{3}{4} + 3\varepsilon k + \frac{9}{2}k^2 + 3\varepsilon k^3 + \frac{3}{4}k^4 - \frac{3}{2}\left(1 + 2\varepsilon k + k^2\right)\omega^2 + \frac{3}{4}\omega^4\right)\rho_\varepsilon \partial_k^2 \rho_t'\Big] - \frac{3}{k}\left(k^2 - \omega^2\right)\rho_0 \rho_l' \\ & + \left(\frac{3}{k} + 2k + \frac{6}{k}\omega - \left(\frac{15}{k^3} + \frac{2}{k}\right)\omega^2 + \frac{18}{k^3}\omega^3\right)\rho_0 \rho_t' + \left(6 - 12\omega\right)\rho_0 \partial_k \rho_l' \\ & + \left(-3 + 6k\omega + \frac{3}{k^2}\omega^2 - \frac{6}{k}\omega^3\right)\rho_0 \partial_k \rho_t' + 3k\rho_0 \partial_k^2 \rho_l' - \frac{3}{2k}\left(k^2 - \omega^2\right)\rho_0 \partial_k^2 \rho_t' \\ & + 12k^2 \epsilon\left(\omega\right)\delta\left(\omega^2 - k^2\right)\rho_l' - 6\left|\omega\right|\delta\left(\omega^2 - k^2\right)\rho_t' - 6k^2\left|\omega\right|\,\partial_{\omega^2}\delta\left(\omega^2 - k^2\right)\rho_l' \,. \end{aligned} \tag{46}$$

It remains to perform the integrals over the frequencies ω and ω' and then over the momentum k. Of course, these integrations are not straightforward and necessitate numerical work [40]. Also, the dimensionless parameter $m(N_c, N_f) = m_g/m_f = \frac{4}{3}\sqrt{\frac{N_c(N_c+N_f/2)}{N_c^2-1}}$ is implicitly present in the spectral densities $\rho_{l,t}$ and so, each case has to be treated separately.

Recall that this direct calculation is performed in the sole context of the Braaten-Pisarski HTL-summed next-to-leading order perturbation. As we emphasized in the introductory remarks, there is the problem of occurrence of infrared divergences to be aware of. Hence, extra work is needed in order to extract these from the finite contributions. One interesting point to wander about is what sort of divergences we will obtain. Indeed, in the direct calculation of $\gamma_l(0)$, the damping rate for longitudinal gluons with zero momentum, the divergent term behaves like $1/\eta^2$ [35]. In the second coefficient in p^2 of $\gamma_t(p)$, the damping rate for transverse gluons, $1/\eta^2$ does also appear together with $\ln \eta$ [37]. The question is then: what sort of divergences will we get for $\gamma_\pm(p)$? If different from $1/\eta^2$ and $\ln \eta$, are we able to understand why?

We stress once again that the occurrence of these divergences may simply be due to the early expansion we make of the HTL-summed next-to-leading order self-energies in powers of the external momentum. We have argued otherwise in section three, but a really more convincing argument would be to carry the very same calculation in a way that avoids such an early expansion. We have indicated that, in the perturbative context, this could be technically very difficult.

In any case, there is by now convincing evidence in the literature that next-to-leading order quantities are magnetic-sensitive. We tend to be of the viewpoint that the occurrence of infrared divergences in HTL-summed next-to-leading order self-energies is probably a manifestation of this magnetic sensitivity, and that a more complete next-to-leading order calculation should remove them. We mean a calculation that takes into account magnetic effects not present at the electric scale, and hence not incorporated in the hard thermal loops. Of course, we may need to understand first such effects. It is then interesting to ask whether the infrared divergences one obtains, in the damping rates and in possibly similar quantities, can be of any help. It may also be that these effects cannot even be grasped perturbatively.

Also, we have argued in section three that regularizing the infrared region with a simple shift in the static transverse gluonic propagator by a momentum-independent magnetic mass may not be the best way to shield from magnetic sensitivity. This is important in view of the fact that much of the results of high temperature QCD rely on such a regularization. In our work, we exclude from the outset the magnetic region by introducing an infrared cutoff of the order of the magnetic scale. One drawback is that our calculation is valid only for very soft external momenta and the results we obtain cannot be carried to larger values. That our results differ analytically from other estimations, particularly those relying on the magnetic-mass shielding, comes mainly from the regularization procedure and its implications. Also, the difference in the results accentuates the sensitivity of the HTL-based perturbation to the magnetic scale and is added evidence that possibly interesting physics is happening between the soft and very soft regions.

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